Chapter 9

Conclusions and Suggestions for Future Research

9.1 Wavelet Transforms

The wavelet transform allows to decorrelate data because wavelets are localized in both space/time and scale/frequency, and because they are based on a multi-resolution analysis. The lifting scheme is an efficient implementation of a wavelet transform algorithm. Furthermore it can easily be converted to operate on integer data. The inverse transform is trivial to find and always provides perfect reconstruction of the input data. We have worked out the details of integer wavelet transforms for a class of biorthogonal wavelets (Cohen-Daubechies-Feauveau).

Not only can the lifting scheme be used to calculate all classical biorthogonal wavelet transforms, it also can be used as a paradigm to design so-called second-generation wavelets: wavelets that are not based on simple translates and dilates of a one-dimensional function. An example of this are the new wavelets we created on a rectangular grid, based on a Red-Black splitting scheme. These Red-Black wavelets are non-separable two-dimensional wavelets, unlike classical tensor-product wavelets. They are less anisotropic and do not favor the horizontal and vertical directions that strongly, as we have shown in a denoising application. Furthermore the idea behind Red-Black wavelets can easily be applied to higher dimensions and more complex geometries.

9.2 A Software Library for Wavelet-Based Image Processing

We developed the C++ software library WAILI (Wavelets with Integer Lifting), which implements wavelet transforms and wavelet-based image processing operations. WAILI is built on the framework of the integer version of the lifting scheme.

The library implements several wavelet transforms belonging to the biorthogonal family of Cohen-Daubechies-Feauveau, on images of arbitrary dimensions. Other wavelet transforms can easily be added, given a decomposition in lifting steps.

WAILI provides image processing operations on wavelet-transformed images. Soft-thresholding using generalized cross validation is an efficient and well-performing method for noise reduction. Other important operations are crop and merge: due to the spatial locality of wavelets, subimages can be cut out or pasted in a larger wavelet-transformed image without retransforming them, while affecting only a limited set of wavelet coefficients.

We also extended WAILI to handle very large images. By tiling the individual wavelet subbands, in a way transparent to the user, large images can be transformed and processed. This is thus done in a block-based manner under supervision of a block manager. The division in blocks also has a positive influence on the performance of the denoising algorithm: since denoising works on the independent blocks, it adapts better to the local properties of the image.

Because most operations require communication of only a limited strip of wavelet coefficients (e.g. lifting steps), or even no communication at all (e.g. denoising), this scheme has potential for parallelization.

9.3 Approximate POD using Wavelet Packets

Due to its excellent decorrelation capabilities, a wavelet packet transform can serve as a good approximation for the Proper Orthogonal Decomposition (POD), which is the theoretical ideal decorrelation method. By compressing the data in the wavelet packet domain, we can dramatically reduce the complexity of the system that is being analyzed.

We used the Approximate Proper Orthogonal Decomposition (APOD) to analyze precalculated solution profiles for a complex dynamical system. We have done experiments with a reaction-diffusion system, described by a partial differential equation (PDE). The APOD allowed a good reconstruction of the original data, requiring only very few eigenmodes, and thus revealed the low-dimensional intrinsics of the dynamical system we are interested in. The APOD can be used to construct a low-dimensional model for the original problem, allowing fast and easy analysis of the dynamical system.

9.4 Suggestions for Future Research

We just suggest two directions for future research:

1. Modern microprocessors have additional instructions for the efficient parallel execution of operations that are heavily used in multi-media applications. Examples are:

   - Altivec from Motorola on the PowerPC,
   - MAX-2 (Multimedia Acceleration eXtensions 2) from Hewlett-Packard on the PA-RISC,
   - MMX (MultiMedia eXtension) from Intel on the Pentium MMX, Pentium II, and Pentium III,
   - MVI (Motion Video Instruction) from Compaq (formerly Digital) on the Alpha AXP,
   - VIS (Visual Instruction Set) from Sun Microsystems on the SPARC V9 architecture.

   Using these instructions, it should be possible to implement a highly optimized version of the integer wavelet transform and plug this into WAILI, yielding a serious performance boost.

2. Currently the extension of WAILI for handling very large images divided in blocks has not been parallelized yet. Modifying the library for parallel execution would be an interesting topic.