Appendix A

Lifting Decompositions

Sample decompositions for some analyzing filter pairs of the family of biorthogonal Cohen-Daubechies-Feauveau wavelets with n ($\widetilde{n}$) vanishing moments for the primal (dual) wavelet [9]. Note that $n = 1, \widetilde{n} = 1$ is the Haar wavelet.

These decompositions are not unique due to the non-uniqueness of the division of Laurent polynomials. We have chosen the factorizations such to preserve symmetry in the lifting steps as much as possible.

n	$\widetilde{n}$	
1		$\widetilde{g}(z) = \frac{1}{2}z^{-1} - \frac{1}{2}$
		$t_1(z) = -1$ $K_1 = 1$
	1	$\widetilde{h}(z) = \frac{1}{2}z^{-1} + \frac{1}{2}$
		$s_1(z) = \frac{1}{2}$ $K_2 = \frac{1}{2}$
	3	$\widetilde{h}(z) = -\frac{1}{16}z^{-3} + \frac{1}{16}z^{-2} + \frac{1}{2}z^{-1} + \frac{1}{2} + \frac{1}{16}z - \frac{1}{16}z^{2}$
		$s_1(z) = -\frac{1}{16}z^{-1} + \frac{1}{2} + \frac{1}{16}z$ $K_2 = \frac{1}{2}$
	5	$\widetilde{h}(z) = \frac{3}{256}z^{-5} - \frac{3}{256}z^{-4} - \frac{11}{128}z^{-3} + \frac{11}{128}z^{-2} + \frac{1}{2}z^{-1} + \frac{1}{2} + \frac{11}{128}z - \frac{11}{128}z^{2} - \frac{3}{256}z^{3} + \frac{3}{256}z^{4}$
		$s_1(z) = \frac{3}{256}z^{-2} - \frac{11}{128}z^{-1} + \frac{1}{2} + \frac{11}{128}z - \frac{3}{256}z^{2}$ $K_2 = \frac{1}{2}$

n	$\widetilde{n}$	
2		$\widetilde{g}(z) = \frac{1}{4}z^{-2} - \frac{1}{2}z^{-1} + \frac{1}{4}$
		$t_1(z) = -\frac{1}{2}z^{-1} - \frac{1}{2}$ $K_1 = 1$
	2	$\widetilde{h}(z) = -\frac{1}{8}z^{-2} + \frac{1}{4}z^{-1} + \frac{3}{4} + \frac{1}{4}z - \frac{1}{8}z^{2}$
		$s_1(z) = \frac{1}{4} + \frac{1}{4}z$ $K_2 = -\frac{1}{2}$
	4	$\widetilde{h}(z) = \frac{3}{128}z^{-4} - \frac{3}{64}z^{-3} - \frac{1}{8}z^{-2} + \frac{19}{64}z^{-1} + \frac{45}{64} + \frac{19}{64}z - \frac{1}{8}z^{2} - \frac{3}{64}z^{3} + \frac{3}{128}z^{4}$
		$s_1(z) = -\frac{3}{64}z^{-1} + \frac{19}{64} + \frac{19}{64}z - \frac{3}{64}z^{2}$ $K_2 = -\frac{1}{2}$
	6	$\widetilde{h}(z) = -\frac{5}{1024}z^{-6} + \frac{5}{512}z^{-5} + \frac{17}{512}z^{-4} - \frac{39}{512}z^{-3} - \frac{123}{1024}z^{-2} + \frac{81}{256}z^{-1} + \frac{175}{256}$ $+ \frac{81}{256}z - \frac{123}{1024}z^{2} - \frac{39}{512}z^{3} + \frac{17}{512}z^{4} + \frac{5}{512}z^{5} - \frac{5}{1024}z^{6}$
		$s_1(z) = \frac{5}{512}z^{-2} - \frac{39}{512}z^{-1} + \frac{81}{256} + \frac{81}{256}z - \frac{39}{512}z^{2} + \frac{5}{512}z^{3}$ $K_2 = -\frac{1}{2}$

n	$\widetilde{n}$	
3		$\widetilde{g}(z) = \frac{-1}{8}z^{-2} + \frac{3}{8}z^{-1} - \frac{3}{8} + \frac{1}{8}z$
		$t_1(z) = 0$
		$s_1(z) = -3z$
		$t_2(z) = \frac{1}{8}z^{-2} + \frac{3}{8}z^{-1}$
		$K_1 = \frac{-1}{2}z^{-1}$
	1	$\widetilde{h}(z) = \frac{-1}{4}z^{-2} + \frac{3}{4}z^{-1} + \frac{3}{4} - \frac{1}{4}z$
		$s_2(z) = -4z^2$
		$K_2 = -z$
	3	$\widetilde{h}(z) = \frac{3}{64}z^{-4} - \frac{9}{64}z^{-3} - \frac{7}{64}z^{-2} + \frac{45}{64}z^{-1} + \frac{45}{64} - \frac{7}{64}z - \frac{9}{64}z^2 + \frac{3}{64}z^3$
		$s_2(z) = \frac{-3}{4}z - 4z^2 + \frac{3}{4}z^3$
		$K_2 = -z$
	5	$\widetilde{h}(z) = \frac{-5}{512}z^{-6} + \frac{15}{512}z^{-5} + \frac{19}{512}z^{-4} - \frac{97}{512}z^{-3} - \frac{13}{256}z^{-2} + \frac{175}{256}z^{-1} + \frac{175}{256} - \frac{13}{256}z - \frac{97}{512}z^2 + \frac{19}{512}z^3 + \frac{15}{512}z^4 - \frac{5}{512}z^5$
		$s_2(z) = \frac{5}{32} - \frac{17}{16}z - 4z^2 + \frac{17}{16}z^3 - \frac{5}{32}z^4$
		$K_2 = -z$

n	$\tilde{n}$	
4		$\tilde{g}(z) = -\frac{1}{16}z^{-3}+\frac{1}{4}z^{-2}-\frac{3}{8}z^{-1}+\frac{1}{4}-\frac{1}{16}z$
		$t_1(z) = 0$ $s_1(z) = -\frac{1}{4}-\frac{1}{4}z$ $t_2(z) = -z^{-1}-1$ $K_1 = 2$
	2	$\tilde{h}(z) = \frac{3}{32}z^{-3}-\frac{3}{8}z^{-2}+\frac{5}{32}z^{-1}+\frac{5}{4}+\frac{5}{32}z-\frac{3}{8}z^{2}+\frac{3}{32}z^{3}$
		$s_2(z) = \frac{3}{16}+\frac{3}{16}z$ $K_2 = -\frac{1}{4}$
	4	$\tilde{h}(z) = -\frac{5}{256}z^{-5}+\frac{5}{64}z^{-4}-\frac{1}{256}z^{-3}-\frac{3}{8}z^{-2}+\frac{35}{128}z^{-1}+\frac{35}{32}+\frac{35}{128}z-\frac{3}{8}z^{2}-\frac{1}{256}z^{3}$ $+\frac{5}{64}z^{4}-\frac{5}{256}z^{5}$
		$s_2(z) = -\frac{5}{128}z^{-1}+\frac{29}{128}+\frac{29}{128}z-\frac{5}{128}z^{2}$ $K_2 = -\frac{1}{4}$
	6	$\tilde{h}(z) = \frac{35}{8192}z^{-7}-\frac{35}{2048}z^{-6}-\frac{55}{8192}z^{-5}+\frac{115}{1024}z^{-4}-\frac{557}{8192}z^{-3}-\frac{733}{2048}z^{-2}+\frac{2625}{8192}z^{-1}$ $+\frac{525}{512}+\frac{2625}{8192}z-\frac{733}{2048}z^{2}-\frac{557}{8192}z^{3}+\frac{115}{1024}z^{4}-\frac{55}{8192}z^{5}-\frac{35}{2048}z^{6}+\frac{35}{8192}z^{7}$
		$s_2(z) = \frac{35}{4096}z^{-2}-\frac{265}{4096}z^{-1}+\frac{499}{2048}+\frac{499}{2048}z-\frac{265}{4096}z^{2}+\frac{35}{4096}z^{3}$ $K_2 = -\frac{1}{4}$

n	$\tilde{n}$	
5		$\tilde{g}(z) = \frac{1}{32}z^{-3} - \frac{5}{32}z^{-2} + \frac{5}{16}z^{-1} - \frac{5}{16} + \frac{5}{32}z - \frac{1}{32}z^{2}$
		$t_1(z) = -5$ $s_1(z) = \frac{1}{40}z^{-1} + \frac{5}{24}$ $t_2(z) = \frac{75}{2} + \frac{45}{2}z$ $K_1 = 15$
	1	$\tilde{h}(z) = \frac{3}{16}z^{-3} - \frac{15}{16}z^{-2} + \frac{5}{4}z^{-1} + \frac{5}{4} - \frac{15}{16}z + \frac{3}{16}z^{2}$
		$s_2(z) = \frac{-1}{75}$ $K_2 = \frac{1}{30}$
	3	$\tilde{h}(z) = \frac{-5}{128}z^{-5} + \frac{25}{128}z^{-4} - \frac{13}{64}z^{-3} - \frac{35}{64}z^{-2} + \frac{35}{32}z^{-1} + \frac{35}{32} - \frac{35}{64}z - \frac{13}{64}z^{2} + \frac{25}{128}z^{3} - \frac{5}{128}z^{4}$
		$s_2(z) = \frac{-1}{360}z^{-1} - \frac{1}{75} + \frac{1}{360}z$ $K_2 = \frac{1}{30}$
	5	$\tilde{h}(z) = \frac{35}{4096}z^{-7} - \frac{175}{4096}z^{-6} + \frac{15}{512}z^{-5} + \frac{25}{128}z^{-4} - \frac{1357}{4096}z^{-3} - \frac{1575}{4096}z^{-2} + \frac{525}{512}z^{-1} + \frac{525}{512} - \frac{1575}{4096}z - \frac{1357}{4096}z^{2} + \frac{25}{128}z^{3} + \frac{15}{512}z^{4} - \frac{175}{4096}z^{5} + \frac{35}{4096}z^{6}$
		$s_2(z) = \frac{7}{11520}z^{-2} - \frac{23}{5760}z^{-1} - \frac{1}{75} + \frac{23}{5760}z - \frac{7}{11520}z^{2}$ $K_2 = \frac{1}{30}$

n	$\tilde{n}$	
6		$\tilde{g}(z) = \frac{1}{64}z^{-4} - \frac{3}{32}z^{-3} + \frac{15}{64}z^{-2} - \frac{5}{16}z^{-1} + \frac{15}{64} - \frac{3}{32}z + \frac{1}{64}z^{2}$
		$t_1(z) = \frac{-1}{6}z^{-1} - \frac{1}{6}$ $s_1(z) = \frac{-9}{16} - \frac{9}{16}z$ $t_2(z) = \frac{-4}{3}z^{-1} - \frac{4}{3}$ $K_1 = 4$
	2	$\tilde{h}(z) = \frac{-5}{64}z^{-4} + \frac{15}{32}z^{-3} - \frac{7}{8}z^{-2} - \frac{7}{32}z^{-1} + \frac{77}{32} - \frac{7}{32}z - \frac{7}{8}z^{2} + \frac{15}{32}z^{3} - \frac{5}{64}z^{4}$
		$s_2(z) = \frac{5}{32} + \frac{5}{32}z$ $K_2 = \frac{-1}{8}$
	4	$\tilde{h}(z) = \frac{35}{2048}z^{-6} - \frac{105}{1024}z^{-5} + \frac{165}{1024}z^{-4} + \frac{235}{1024}z^{-3} - \frac{1827}{2048}z^{-2} + \frac{63}{512}z^{-1} + \frac{987}{512} + \frac{63}{512}z - \frac{1827}{2048}z^{2} + \frac{235}{1024}z^{3} + \frac{165}{1024}z^{4} - \frac{105}{1024}z^{5} + \frac{35}{2048}z^{6}$
		$s_2(z) = \frac{-35}{1024}z^{-1} + \frac{195}{1024} + \frac{195}{1024}z - \frac{35}{1024}z^{2}$ $K_2 = \frac{-1}{8}$
	6	$\tilde{h}(z) = \frac{-63}{16384}z^{-8} + \frac{189}{8192}z^{-7} - \frac{119}{4096}z^{-6} - \frac{777}{8192}z^{-5} + \frac{1101}{4096}z^{-4} + \frac{557}{8192}z^{-3} - \frac{3465}{4096}z^{-2} + \frac{2079}{8192}z^{-1} + \frac{14091}{8192} + \frac{2079}{8192}z - \frac{3465}{4096}z^{2} + \frac{557}{8192}z^{3} + \frac{1101}{4096}z^{4} - \frac{777}{8192}z^{5} - \frac{119}{4096}z^{6} + \frac{189}{8192}z^{7} - \frac{63}{16384}z^{8}$
		$s_2(z) = \frac{63}{8192}z^{-2} - \frac{469}{8192}z^{-1} + \frac{843}{4096} + \frac{843}{4096}z - \frac{469}{8192}z^{2} + \frac{63}{8192}z^{3}$ $K_2 = \frac{-1}{8}$